

Adult Education Program

Math I - Applied Math



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Whole Numbers and Operations

Basic to solving all math problems is a good, strong foundation in whole number math facts, mainly the addition facts and multiplication tables. If you are unsure of any of these facts, now is the time to review and memorize those that give you trouble. Just as you can remember the phone numbers, addresses, and other important numbers in your life, you can remember these facts. Your ability to work with numbers depends on how quickly and accurately you can do these computations.

Before starting work on problem solving, let's review the basic operations of addition, subtraction, multiplication, and division with whole numbers. The examples below will help refresh your memory about the basics of these operations. An estimate for each problem is also shown.

Addition

Example 1 shows how to regroup (carry) in some addition problems. Don't forget that numbers can be added in any order without affecting the answer.

Example 1

$$486 + 39 + 1725 + 89 = 2339$$

$$\begin{array}{r}
 1 \ 2 \ 2 \leftarrow \text{Regroup to the next column to the left.} \\
 486 \\
 39 \\
 1725 \\
 + \quad 89 \\
 \hline
 = 2339
 \end{array}$$

$$\begin{array}{r}
 \text{Estimate:} \quad 500 \\
 \quad \quad \quad 50 \\
 \quad \quad \quad 1700 \\
 + \quad \quad 100 \\
 \hline
 = 2350
 \end{array}$$

Subtraction

Example 2 illustrates regrouping (borrowing) in subtraction problems. Remember that the number following the subtraction sign (-) is the number subtracted and placed on the second line.

Example 2

$$4006 - 957 = 2339$$

$$\begin{array}{r}
 4006 \leftarrow \text{Regroup from the 400 by reducing it to 399} \\
 - 957 \\
 \hline
 = 3049
 \end{array}$$

adding 10 to the 6, which makes it 16.

$$\begin{array}{r}
 \text{Estimate:} \quad 4000 \\
 - 1000 \\
 \hline
 = 3000
 \end{array}$$

Multiplication

In example 3 shows both regrouping and indenting in multiplication problems. Numbers can be multiplied in any order without affecting the answer.

Example 3

$$48 \times 32 =$$

Division

For most people, division is the most complicated operation. Example 4 shows the basic steps. Remember that the number following the division symbol () is the divisor.

Example 4

$$3049 \div 6 =$$

STEP 1 Divide by the number that follows the division sign (\div). Divide into the first number that you can. Since 6 won't divide into 3, divide into 30.

STEP 2 Multiply and subtract. Bring down the next number. If you can't divide into it, write a zero in the answer before you bring down the next number.

$$\begin{array}{r} 50 \\ 6\overline{)3049} \\ \underline{-30} \\ 04 \end{array}$$

STEP 3 Divide 6 into 49. Multiply. Subtract. Write the **remainder** as a part of your answer.

$$\begin{array}{r} 508\text{ r}1 \\ 6\overline{)3049} \\ \underline{-30} \\ 049 \\ \underline{-48} \\ 1 \end{array}$$

Estimate: $3000 \div 6 = 500$

$$\begin{array}{r} 500 \\ 6\overline{)3000} \end{array}$$

There is no one correct estimate. Use numbers that are easy to work with when estimating. Estimates will vary. Your calculator can be used to perform basic whole number operations. Become familiar with the keys on your calculator and how to use them.

Start Key	ON AC C	Turns the calculator on Clears all numbers and operations, displays 0. Clears only the last number or operation entered.
Operation keys	+ - x ÷	Addition Subtraction Multiplication Division
Number keys	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	
Equal sign	=	

Practice using your calculator. Be sure to estimate a reasonable answer to each problem you do. This will be a good way for you to determine whether or not your calculator answer is reasonable. Whether you work with or without a calculator, you are responsible for your answers. It is easy to hit the wrong key or to enter numbers in the wrong order. Your estimate and common sense will help you recognize when this happens.

To Perform an Operation

PROCESS	KEY IN #	DISPLAY	EXAMPLE: 12 + 3 = 4
Step 1	AC to clear the display	0.	0.
Step 2	first number	first number	12.
Step 3	operation symbol (÷)	first number	12.
Step 4	second number	second number	3.
Step 5	equal sign (=)	answer	4.

EXERCISE 1

Whole Number Operations

Directions: Solve each problem using paper and pencil. Find an estimate and an exact answer for each problem. Compare the two answers. Are they close? After you finish each problem, check your answer with your calculator.

1.
$$\begin{array}{r} 25 \\ + 64 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 425 \\ + 46 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 7348 \\ + 4385 \\ \hline \end{array}$$

4. $578 + 36$

5. $356 + 12 + 477$

6. $228,347 + 6,287$

7.
$$\begin{array}{r} 574 \\ - 362 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 4383 \\ - 2227 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 348,000 \\ - 36,549 \\ \hline \end{array}$$

10. $2,860 - 644$

11. $712 - 99$

12. $5,000 - 879$

13.
$$\begin{array}{r} 43 \\ \times 3 \\ \hline \end{array}$$

14.
$$\begin{array}{r} 425 \\ + 46 \\ \hline \end{array}$$

15.
$$\begin{array}{r} 7348 \\ + 4385 \\ \hline \end{array}$$

16. $4,000 \times 30$

17. 258×64

18. $4,230 \times 16$

19. $7 \overline{)287}$

20. $9 \overline{)5643}$

21. $15 \overline{)4050}$

22. $5418 \div 6$

23.
$$\begin{array}{r} 1200 \\ \underline{60} \end{array}$$

24. $7448 \div 16$

Problem Solving

In this section, we will discuss five steps you can use to solve word problems. Most of the problems on the Mathematics Test are word problems that you can solve more skillfully if you practice these five steps.

5 STEPS FOR SOLVING WORD PROBLEMS

- | | |
|---|----------------------------------|
| 1. Understand what the question is asking. | -Question |
| 2. Organize data and identify information necessary to solve the problem. | -Information |
| 3. Select a problem-solving strategy using mathematical operations. | -Operation |
| 4. Set up the problem, estimate, and then compute the exact answer. | -Setup & Estimation; Computation |
| 5. Check the reasonableness of the answer. | -Is the answer reasonable |

STEP 1 Understand what the question is asking. Read the problem thoroughly and then determine exactly what the question is asking. In the example below the question is in italic type.

Example Two pancakes contain 120 calories. *How many calories did Fred consume* if he had a stack of 6 pancakes for breakfast?

STEP 2 Organize the data and identify information necessary to solve the problem. Choose only the information needed to solve the problem. As in real life, there is often more information than you need. Once in a while there may not be enough information to solve the problem. Select numbers with their unit labels. In the examples below the question is in italic type and the necessary information is underlined.

Example 1 In the precinct, only 700 people are registered voters out of an adult population of 1200. If 300 voters are women, how many voters are men?

(In this problem the number 1200 is not needed to solve the problem.)

Example 2 At the 20% off sale, Alan bought 6 ties. How much did he pay?

(There is not enough information given to solve the problem because the original price of the ties is not given.)

STEP 3 Select a problem-solving strategy using appropriate mathematical operations. After you have determined the question and identified the necessary information, you will select a problem-solving strategy. Each problem will contain key words or concepts that will help you decide what to do. Many times, your intuition will help you determine which arithmetic operation to choose. This intuition comes from both your personal experience and your understanding of math concepts. Remember, there are only four operations from which to choose: addition, subtraction, multiplication, and division. The following chart will help you decide which operation to use.

ADD

1. When *combining* amounts to get a larger number
2. When finding a *total*

The order in which numbers are added makes no difference in addition.

For example: $2 + 4 = 6$ and $4 + 2 = 6$

DIVIDE

1. When *given* the amount for *several* equal items and *asked to find* the amount for one item
2. When *splitting, cutting, sharing into equal parts*
3. When asked *how many* times some quantity goes into another
4. When asked to *find an average*

There are three ways to show division.

$$8 \div 4 = 2$$

$$\frac{8}{4} = 2$$

$$4 \overline{)8}$$

Remember that the order of the numbers makes a difference in division. The number you are dividing by must be the divisor. For example:

$$\frac{8}{4} = 2$$

but $\frac{4}{8} = \frac{1}{2}$

SUBTRACT

1. When taking away an amount to obtain a smaller number
2. When finding the difference between two amounts

Keep in mind that the order of the numbers makes a difference in subtraction. The number following the minus sign is the number subtracted.

For example: $6 - 4 = 2$ but $4 - 6 = -2$

MULTIPLY

1. When *given one* unit of something and *asked to find* the total for *several*
2. When asked to *find a fraction* of a quantity
3. When asked to *find a percent* of a quantity

There are three ways to show multiplication.

$$2 \times 4 = 8 \quad 2 \cdot 4 = 8 \quad 2(4) = 8$$

The order in which numbers are multiplied makes no difference in multiplication.

For example: $2 \times 4 = 8$ and $4 \times 2 = 8$

Example 1 How *many miles did Luis get per gallon* when he drove 300 miles on 15 gallons of gas? (**Divide** because you are asked to find the number of miles for one gallon when you are given the miles for 15 gallons.)

Example 2 Sandy lost 34 pounds on her diet. *What was her original weight*, if her weight is now 123 pounds? (**Add** because you are asked to combine her current weight with the pounds she lost to get her original weight.)

Example 3 Ali's monthly salary is \$2500. *What is his yearly salary*? (**Multiply** because you are given one month's salary and asked to find the salary for several months. Notice that you will need to know there are twelve months in a year to solve the problem.)

STEP 4 **Set up the problem, estimate the answer, and then compute the exact answer.** After choosing the operation, set up the problem. (In some problems, you may be asked only to identify the correct setup and not have to proceed any further.) Next, estimate an answer using compatible, rounded numbers. For some problems an estimate can be the final answer, as when a question asks for an approximate or rounded value. At other times, you can use the estimate to check whether you have chosen the right setup and whether your answer makes sense. The estimate is especially important if you are using a calculator; the estimate can help you check to make sure you entered the numbers and symbols correctly on the calculator. After estimating the answer, solve the problem using the exact information and operations you've set up. Do the computations carefully and accurately.

STEP 5 **Check the reasonableness of the answer.** The final step in solving any problem is to evaluate your answer. Does your solution answer the question? Is your answer a sensible one? Is your calculation close to your estimate?

Let's go through the five problem-solving steps for the next two examples.

Example 1 On the first day of your 750-mile road trip, you drove 448 miles in 7 hours. What was your average rate of speed?

Question	What was your average speed?
Information	448 miles; 7 hours
Operation	Average means divide
Setup and Estimation	420 miles ÷ 7 hours = 60 miles per hour (estimate)
Computation	448 miles ÷ 7 hours = 64 miles per hour (exact)
Is the answer reasonable?	Yes, because 64 miles per hour is a reasonable speed on the highway and it is very close to the estimate.

Example 2 How many miles did Luis get per gallon when he drove 300 miles on 15 gallons of gas? (Divide because you are asked to find the number of miles for one gallon when you are given the miles for 15 gallons.)

Question	What is the secretary's annual pay?
Information	%572 per week; 52 week in one year
Operation	Multiply to find the total for 52 weeks when given one week.
Setup and Estimation	\$600 x 50 weeks - \$30,000 annually (estimate)
Computation	\$572 x 52 weeks - \$29,744 annually (exact)
Is the answer reasonable?	Yes, because \$29,744 is a reasonable salary and it is close to the estimate.

Look at Example 2 above. Suppose you had misunderstood the problem and had divided instead of multiplying: $\$572 \div 52 = \11 . This wrong answer tells you that the secretary earned \$11 in a year. Would it make sense for the secretary to earn less in a year than in a week? No. By checking whether your answer makes sense, you may be able to catch a mistake.

EXERCISE 2

Problem Solving

Directions: The following problems will give you a chance to practice the 5-step approach to problem solving. For each problem fill in the chart.

1.	Question	Understand what the question is asking
2.	Information	Organize data and identify information necessary to solve the problem.
3.	Operation	Select a problem-solving strategy using mathematical operations.
4.	Setup and Estimation Computation	Set up the problem, estimate, and then compute the exact answer.
5.	Is the answer reasonable?	Check the reasonableness of the answer.

1. As you leave for a three-day mini-vacation, the odometer on your car reads 49,752 miles. If you drive 162 miles to get to your destination and the same distance to return home, what will your mileage on the odometer read at the end of the trip?
 (1) 324 (2) 49,428 (3) 49,590 (4) 49,914 (5) 50,076
2. Michele and her three coworkers shared the winning lottery ticket. How much was each person's share if the ticket was worth \$12,000,000?
 (1) \$2,000,000 (2) \$3,000,000 (3) \$4,000,000 (4) \$6,000,000
 (5) Not enough information given.
3. Approximately how many square miles larger is Canada than the United States? The United States is 3,675,633 square miles, and Canada is 3,851,809 square miles.
 (1) 17,000 (2) 18,000 (3) 170,000 (4) 180,000 (5) 1,800,000
4. A catalog listed sale prices for three different models of 19-inch TVs: \$199, \$249, and \$189. If the buyer for the model purchased 97 TVs at the lowest price, what is the total cost?
 (1) \$286 (2) \$637 (3) \$18,333 (4) \$19,303 (5) \$24,153
5. Marta is self-employed and pays \$2496 quarterly for estimated tax. How much should she put aside for tax payment each month if a quarter is three months?
 (1) \$624 (2) \$832 (3) \$7,488 (4) \$9,984 (5) \$29,952
6. Jose makes \$2000 a month. He pays \$425 for a month's rent and \$90 per week for food. How much does he have left each month after paying for rent?
 (1) \$515 (2) \$1485 (3) \$1575 (4) \$1910 (5) 2425
7. If an employer gives each employee a \$500 end-of-year bonus, what is the total cost of the bonus program for 25 employees?
 (1) \$25 (2) \$475 (3) \$525 (4) \$10,000 (5) \$12,500

Multistep Problems

Many of the problems you see on the Mathematics Test will take more than one calculation to solve. As with problems you've already done, use the 5-step process, but identify all the steps you need to take.

Example The yearly rainfall for four consecutive years in Rock Falls was 22 inches, 28 inches, 31 inches, and 19 inches. Find the average rainfall over the four-year period.

Question	What is the average rainfall?
Information	22 in., 28 in., 31 in., and 19 in.; 4 years
Operation	Step 1: To get the total rainfall – ADD Step 2: Find the average means – DIVIDE
Setup and Estimation	$20 + 30 + 30 + 20 = 100$ inches (<i>setup</i>) $100 \div 4 = 25$ inches (<i>estimate</i>)
Computation	$22 + 28 + 31 + 19 = 100$ inches (<i>setup</i>) $100 \div 4 = 25$ inches (<i>exact</i>)
Is the answer reasonable?	Yes, because the average is within the range of 19 to 31 inches. The e calculation are not only close; they are an exact match!

With multistep problems, after you have finished your calculations, go back and see if your answer to the question makes sense. If you have forgotten a step, this can help you catch yourself and finish the problem correctly.

Personalizing a Problem

If you have trouble restating a problem, reread it several times. Think about how you might solve a similar problem in your own life. Many of the problems on the EIPCS Test will involve questions about practical, everyday situations.

Example The total cost of a computer system is \$3210. A down payment of \$1500 is made, and the balance is to be paid in 18 equal monthly payments. Find the monthly payments.

As you read this problem, recall the times you have purchased something and put down a deposit, planning to pay the rest later. You subtracted the deposit from the purchase price to find out how much you still owe.

\$3210 purchase price —1500 deposit
\$1710 amount owed

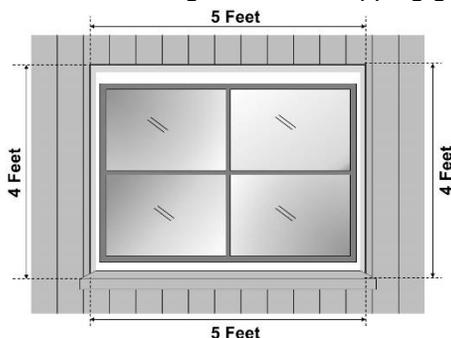
According to the problem, you can pay the amount owed in 18 equal payments. So you have to divide by 18 to find out how much you will pay each month.

$$\$170 \div 18 = \$95 \text{ per month}$$

Restarting the Problem Using Words, Sketches, or Diagrams

It often helps to restate the problem by organizing the information in a way that puts the problem in your own words. You might actually talk to yourself. As you do this, jot down notes, sketches, or diagrams that help you organize your thoughts.

Example 1 To better insulate his house, Miguel had to put new weather stripping around the large window in his family room. The window frame measured 5 feet wide and 4 feet high. How many feet of weather stripping did he need for that window?
First sketch the window and label the lengths of the 4 sides. Remember that the opposite sides of the window are the same length and the stripping goes around the entire window.



This sketch helps you see that you have to add all the sides

Miguel will need 18 feet of weather stripping: $5 + 5 + 4 + 4 = 18$

Example 2 John had a checking account balance of \$425 before he deposited \$187. After he made this deposit, he wrote two checks—one to the grocery for \$43 and another to the cleaners for \$21. Find the checkbook balance after he completed these transactions.

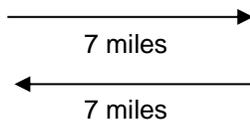
Make a chart to show these transactions.

Amount in Account
\$425
+ \$187
<hr/>
\$612

Checks Written
\$425
+ \$187
<hr/>
\$612

Checkbook Balance
\$425
+ \$187
<hr/>
\$612

Example 3 The math teacher drives 7 miles each way to school. How many miles does the teacher drive to and from school during a five-day workweek?
 Draw a sketch to show the teacher's route each day. Remind yourself that the teacher does this five times per week.



The round trip is $7 + 7 = 14$ miles.

The teacher makes this roundtrip drive 5 times per week, so the total driving would be $5 \times 14 = 70$ miles.

Number Sense

In mathematics there are rules and procedures that guide every operation. It is essential to become familiar with the rules and the appropriate application of the rules. This is an important part of number sense. There are special vales for the use of symbols—such as parentheses, exponents, and radicals—and for the order of all operations performed. Good problem solving will require you to use your number sense and mathematical knowledge effectively. In this chapter you'll look at these topics and learn how to use this information to evaluate formulas.

Powers

When a number is multiplied by itself, we say that it is squared. We show that with a small "2" placed to the upper right of the number.

For example: $7^2 = 7 \times 7 = 49$.

This means seven squared is 7 times 7, which is 49.

In the expression 7^2 , the two is called the power or exponent, and the seven is called the base. It is best to think of the exponent as an instruction. The exponent tells you what to do with the base. When the exponent is two, the base is squared, and we multiply the base by itself.

Examples	$3^2 = 3 \times 3 = 9$	$9^2 = 9 \times 9 = 81$
	$10^2 = 10 \times 10 = 100$	$(\frac{1}{3})^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Sometimes the exponent is a number other than 2.

The exponent tells how many times to multiply the base by itself.

For example:

$$3^4 = 3 \times 3 \times 3 \times 3 = 81$$

$$5^3 = 5 \times 5 \times 5 = 125$$

$$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$$

$$(\frac{2}{3})^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$$

SPECIAL RULES ABOUT POWERS

1. The number 1 to any power is 1. $1^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$
2. Any number to the first power is that number. $6^1 = 6$
3. Any number to the zero power is 1. $14^0 = 1$

When you multiply a number by itself, you get a perfect square. For instance, $6^2 = 36$; so 36 is a perfect square. Some perfect squares that are helpful to know are listed here.

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$20^2 = 400$
$2^2 = 4$	$7^2 = 49$	$12^2 = 141$	$30^2 = 600$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$40^2 = 1600$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$50^2 = 2500$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$100^2 = 10,000$

Square Roots

The operation opposite of squaring a number is finding the square root of a number. The symbol for square root is the radical symbol, $\sqrt{\quad}$. For instance, $5^2 = 25$, so $\sqrt{25} = 5$. $\sqrt{25}$ is read, "the square root of 25." Being familiar with perfect squares makes many square roots easy to recall. For instance, $\sqrt{100} = 10$ because $10^2 = 100$.

If you need to find the square root of a number that is not a perfect square, you can do one of three things: simplify the square root, approximate the square root, or use a calculator.

Example 1 Simplify a square root by writing the number as a product of numbers using perfect squares if possible. $\sqrt{75}$ can be expressed as $= \sqrt{75} \times 3 = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$. You choose 25 and 3 because 25 is a perfect square and you can find its square root.

Example 2 Approximate the square root of a number by looking at the perfect square closest to the number to make an estimated guess at the answer.

$\sqrt{75}$ is between $\sqrt{64}$ and $\sqrt{81}$ so,
 $\sqrt{64} = 8$
 $\sqrt{75} \approx 8.7$ ($\sqrt{75}$ is between 8 and 9 but closer to 9, so we estimate about 8.7.)
 $\sqrt{81} = 9$

Remember always, $x \approx y$ means x is approximately equal to y . This may also be written \simeq , \cong , \sim , $\u2264$ (Libra Symbol), or \approx .

Your calculator can be used to perform powers and roots. Become familiar with these keys on your calculator and how to use them.

Keys:	x^2	squares the numbers
	x^y	raises the number to the power indicated
	SHIFT	accesses key operations indicated above the key
	$\sqrt{\quad}$	finds the square root of the number

To Find Powers and Roots

Process	Key In:	Display	Example: $5^2 = 25$
Step 1	AC to clear the display	0.	0.
Step 2	base	base	5.
Step 3	square symbol (x^2)	answer	25.
Process	Key In:	Display	Example: $5^4 = 625$
Step 1	AC to clear the display	0.	0.
Step 2	base	base	5.
Step 3	power symbol (x^y)	base	5.
Step 4	exponent	power	4.
Step 5	equals sign (=)	answer	625.
Process	Key In:	Display	Example: $\sqrt{36} = 6$
Step 1	AC to clear the display	0.	0.
Step 2	number	number	36.
Step 3	SHIFT	number	36.
Step 4	square root symbol ($\sqrt{\quad}$)	answer	6.

EXERCISE 3

Powers and Roots

Directions: Evaluate the powers below. Use your calculator to check your answer.

- | | | | | |
|-----------|----------|----------|-----------|-----------|
| 1. 8^2 | 2. 9^3 | 3. 1^7 | 4. 2^4 | 5. 4^0 |
| 6. 10^3 | 7. 0^2 | 8. 3^3 | 9. 12^2 | 10. 5^4 |

Evaluate the square roots below. When finding the square root of a perfect square, give the exact answer. When finding a square root that is not a perfect square, give either an approximate answer or a simplified answer. Use your calculator to check your answers.

- | | | | | |
|------------------|-----------------|------------------|-----------------|-----------------|
| 11. $\sqrt{81}$ | 12. $\sqrt{9}$ | 13. $\sqrt{1}$ | 14. $\sqrt{50}$ | 15. $\sqrt{32}$ |
| 16. $\sqrt{144}$ | 17. $\sqrt{28}$ | 18. $\sqrt{400}$ | 19. $\sqrt{0}$ | 20. $\sqrt{12}$ |

Parentheses

Parentheses are powerful symbols in mathematics problem solving. As you know, parentheses can be used to indicate multiplication. For example, $5(3) = 15$. Parentheses can also be used as a grouping symbol to organize a problem. For instance, $5(3 + 6) = 45$. Before you multiply by 5, you should add 3 and 6 because they are grouped together inside the parentheses. Use parentheses anytime you want to emphasize an operation and want that operation to precede other operations. As you practice the order of operations described below, you will see how the parentheses impact the answer to a problem.

Order of Operations

Accurate calculations depend on careful use of addition, subtraction, multiplication, and division. These operations must be performed in a certain order when there are two or more operations in the same problem. The rules that describe that order are called the order of operations. All operations should be performed moving from the left to the right. Be sure to start at the beginning for every problem. Do not skip a step unless there is no operation to do at that level.

5 STEPS to the ORDER OF OPERATIONS

First, do all the work grouped inside parentheses or above a fraction bar.

Second, evaluate powers and square roots.

Third, multiply and divide as indicated from left to right.

Last, add and subtract as indicated from left to right.

Example 1

Calculate the expression	$5 + 7 \times 3$.
First, notice there are no parentheses in the problem. Then multiply 7	$5 + 7 \times 3 =$
Last, add $5 + 21$ to get 26.	$5 + 21 = 26$

Example 2

Calculate	$3(5 + 7)$
First, add the numbers inside the parentheses.	$3(5 + 7) =$
Then multiply 3×12 to give you 36	$3 \times 12 = 36$

Example 3

Calculate

$$\frac{5 + 7 + 6}{3}$$

First, add the numbers grouped above the fraction bar.

$$\frac{5 + 7 + 6}{3}$$

Then divide the sum by 3 to get 6.

$$\frac{18}{3} = 6$$

Example 4

Calculate

$$3 \times 5 + 2 \times 10.$$

First, notice there are no parentheses.

Then multiply 3×5 and 2×10 .

Last, add $15 + 20$ to get 35.

$$15 + 20 = 35$$

Example 5

Calculate

$$4 \times 5^2.$$

First, notice there are no parentheses.

$$4 \times 5^2 =$$

Then raise 5 to the second power.

Last, multiply 4×25 to get 100.

$$4 \times 25 = 100$$

The calculator that you will use on the EIPCS Test has the order of operations programmed in it. If a problem has more than one operation, the calculator will follow the order of operations to arrive at the answer. Be sure to key in the problem correctly.

Keys: [(open parentheses
)] close parentheses

To evaluate a numerical expression on a calculator, key in the numbers, symbols, and operations as they occur. Use the open *parentheses* and close *parentheses* keys to indicate an operation that is to be calculated first. If a number precedes the parentheses, it indicates that you are to multiply the amount in the parentheses by the number.

For example: $5(3 + 8)$ means $5 \times (3 + 8)$

You must key in the multiplication sign.

EXERCISE 5

Order of Operations

Directions: Evaluate each numerical expression below without using a calculator. Be sure to follow the order of operations rules. Then use a calculator to check your answers.

- | | | | |
|--------------------------------|--------------------------|----------------------------|---------------------------|
| 1. $5 + 9 \times 3 =$ | 2. $(4 + 9) \times 5 =$ | 3. $12 - 3 - 2 =$ | 4. $2 + 6 \times 4 + 8 =$ |
| 5. $12 - (7 + 4) =$ | 6. $12 - 7 + 4 =$ | 7. $\frac{5 + 7 + 6}{3} =$ | 8. $\frac{6}{3} + 9 =$ |
| 9. $8 \times 3 + 6 \times 4 =$ | 10. $4 \times (8 - 3) =$ | 11. $4 \times 8 - 3 =$ | 12. $30 + 5 \times 2 =$ |
| 13. $7 - 5 + 3 - 5 =$ | 14. $6 + 21 + 3 - 5 =$ | 15. $15 - 3 + 2^2 =$ | 16. $4(5 + 3)^2 =$ |

Setup Problems

Some problems on the EIPCS Test will require you to identify the correct setup to solve them. These setup problems do not require you to perform the calculations. You are being tested on your ability to demonstrate how you can use numbers, operations, and mathematical processes to solve problems.

To set up a problem, represent the mathematical relations using numbers and operation symbols. This relationship is called an arithmetic expression. There are rules that must be followed when writing an arithmetic expression involving more than one arithmetic operation.

WRITING ARITHMETIC EXPRESSIONS

1. Write an arithmetic expression using numbers and operation symbols following the rules of operations and identifying the individual steps in the multistep problem.
2. Use parentheses or the fraction bar to separate one part of an arithmetic expression from another.
3. Follow the **order of operations** rules to indicate the order in which the arithmetic expression should be solved.

First, do the operations grouped inside parentheses or above the fraction bar. Second, multiply and divide as indicated from left to right. Last, add and subtract as indicated from left to right.

Example

If Becky can walk a mile in 20 minutes, how far can she walk in 3 hours?

Question: How far does she walk in 3 hours?

Information: One mile in 20 minutes; 3 hours, 60 minutes in one hour

Operations: Step 1: To get the total minutes—multiply 3×60
Step 2: To get the number of miles—divide by 20

Setup and Estimation: $\frac{5 + 7 + 6}{3}$ This is the arithmetic expression that sets up the problem

Computation: $3 \times 60 = 180$, $180/20 = 9$ miles

Is the answer reasonable? Yes, because one mile in 20 minutes would be 3 miles per hour $\times 3$ hours = 9.

EXERCISE 6

Arithmetic Expressions

Directions: In each of the following problems, select the arithmetic expression that represents the appropriate setup to solve the problem.

1. in 2001 Harper College charged \$55 per credit hour. Find the total bill for a student who takes 15 credit hours and pays a \$75 activity fee.

- (1) $55 + 75 + 15$
- (2) $75 + 15 \times 55$
- (3) $(75 + 55) \times 15$
- (4) $75 \times 55 \times 15$
- (5) $55 \times 15 - 75$

2. If Kelly buys a dozen oranges @ 45c each and eight apples @ 50c each, how much is his total bill?

- (1) $12 + .45 + 8 + .50$
- (2) $12 \times .45 \times 8 \times .50$
- (3) $(12 + 8) \times (.45 + .50)$
- (4) $12 \times .45 + 8 \times .50$
- (5) $(12 + .45) \times (8 + .50)$

3. Angela used her 75c-off coupon to buy a roll of film for her camera. If the film was priced at \$3.98 and tax was 31c, how much change did she receive if she paid with a \$5 bill?

- (1) $5.00 - (3.98 + 0.31 - 0.75)$
- (2) $5.00 - 3.98 - 0.31 - 0.75$
- (3) $3.98 + 0.31 + 0.75 - 5.00$
- (4) $3.98 + 0.31 - 0.75 - 5.00$
- (5) $5.00 + 3.98 + 0.31 + 0.75$

4. As a waiter Jorge was responsible for four tables. If his customers left tips of \$5, \$7, \$3, and \$9, what was the average tip per table?

- (1) $5 + 7 + 3 + 9$
- (2) $4 \times (5 + 7 + 3 + 9)$
- (3) $\frac{5 + 7 + 6}{3}$
- (4) $5 + 7 + 3 + 9 \div 4$
- (5) $4 \div (5 + 7 + 3 + 9)$

EXERCISE 7

Word Problems

Directions: Solve each problem. Check your work using your calculator.

1. You just inherited an apartment building. If your income from apartment rentals is \$5160 per month, what is your income?

- (1) \$ 430
- (2) \$ 5,160
- (3) \$15,480
- (4) 1)761,920
- (5) Not enough information is given

2. There were 395 people who registered for a sightseeing tour. If a tour bus can hold 72 people, how many buses will be needed to accommodate everyone?

- (1) 5
- (2) 6
- (3) 7
- (4) 10
- (5) 50

3. A garment factory completed an order for 500 pairs of pants and was paid \$5500. If the factory received \$9130 for a second order for pants at the same price, how many pairs of pants were in the second order?

- (1) 363
- (2) 500
- (3) 830
- (4) 913
- (5) 3630

4. A charity organization collected \$348,224 in a recent fundraising drive. Of the amount collected, some was put aside to pay expenses. The remaining money was divided equally among six local charities. What amount did each charity receive?

- (1) \$ 58,041
- (2) \$ 58,047
- (3) \$ 58,053
- (4) \$348,284
- (5) Not enough information is given.

Questions 5-7 refer to the information in the table below.

The local school district has experienced a recent increase in enrollment. The table compares enrollment figures between 1992 and 2002 for its five schools.

High School Student Enrollment

Year	<i>Lincoln</i>	<i>Mead</i>	<i>Sandburg</i>	<i>Austin</i>	<i>Edison</i>
1992	1420	1650	1847	1296	1318
2002	1686	1982	2234	1648	1846

5. Which school had the greatest increase in enrollment?
 - (1) Lincoln
 - (2) Mead
 - (3) Sandburg
 - (4) Austin
 - (5) Edison

6. What was the total increase in enrollment in the district during the ten-year period?
 - (1) 1865
 - (2) 2865
 - (3) 7531
 - (4) 9396
 - (5) Not enough information is given.

7. What was the average increase in enrollment per school in the district?
 - (1) 373
 - (2) 1865
 - (3) 1879
 - (4) 7531
 - (5) 9396

8. Four technicians for Transcom must produce 1298 parts in a week's time. The first one produces 350 parts, the second produces 375 parts, and the third produces 417 parts. Which expression shows how many parts remain for the fourth technician to make?
 - (1) $1298 + 4$
 - (2) $350 + 375 + 417$
 - (3) $1298 - (350 + 375 + 417)$
 - (4) $(350 + 375 + 417) - 1298$
 - (5) $(350 + 37 + 417) + 3$

9. Gary bought a new sport-utility vehicle. The price was \$19,700 including an AM/FM CD/cassette radio and automatic transmission. The car dealer gave him a \$500 rebate for the purchase of the new car, and Gary made a down payment of 53150. What would be the expression for his monthly payments if he takes 60 months to pay the remaining balance?
 - (1) $19700 + 3150 + 500 + 60$
 - (2) $60 + (19700 + 3150 + 500)$
 - (3) $\frac{19700 - (360150 + 500)}{60}$
 - (4) $\frac{19700 - 360150 + 500}{60}$
 - (5) $60 \times (19700 + 3150 + 500)$

10. The manager of a men's shop can buy 80 men's suits for \$4800 and a variety of sports coats at \$43 each. if she decides to buy the 80 suits and 25 coats, which expression shows how to find the total cost of the order?

- (1) 25×43
- (2) $25 + 43 + 4800$
- (3) $25 \times 43 \times 4800$
- (4) $4800 + 25 \times 43$
- (5) $(4800 + 25) \times 43$

Formulas

Letters of the alphabet often are used to represent numbers that you need to find. For instance, you may remember the distance formula $d = rt$. The letters d, r, and t are used to represent numbers for distance, rate, and time. Letters used this way are called unknowns or variables. This use of letters helps us express general relationships about numbers.

Formulas are a way of showing these general relationships. Some common formulas that help us are the area of a circle ($A = \pi r^2$), the perimeter of a rectangle ($P = 2l + 2w$), and the Pythagorean Theorem ($a^2 + b^2 = c^2$). The EIPCS Mathematics Test will include a formula page that will help you solve some problems on the Test. As you read a particular problem, you will have to decide which formula will help you solve it. You should become familiar with the formula page and practice using it as you solve problems. It is located on page 13.

Evaluating Formulas

When you replace letters with numbers in a formula, you substitute numbers for letters. When you perform mathematical operations on the substituted values, you are evaluating a formula. When you evaluate a formula, be sure to follow the order of operations.

Example 1 The formula for the perimeter of (distance around) a rectangle is $P = 2l + 2w$. Find the perimeter of a rectangle if the length is 6 and the width is 5.

STEP 1	Substitute in the formula.	$P = 2l + 2w$
		$P = 2(6) + 2(5)$
STEP 2	Multiply first.	$P = 12 + 10$
STEP 3	Add.	$P = 22$

Example 2 Find the volume of a rectangular box whose length is 7 inches, width is 5 inches, and height is 3 inches. Use the formula for volume $V = lwh$ (see the formula on page 922).

STEP 1	Find the formula for volume.	$V = lwh$
STEP 2	Substitute the values given.	$V = (7)(5)(3)$
STEP 3	Multiply as indicated by the parentheses.	$V = 105$ cubic inches

EXERCISE 8

Evaluating Formulas

Directions: Select the appropriate formula from the formula page. Substitute the values given and evaluate the formula. The units for your answers are given in parentheses following the problem.

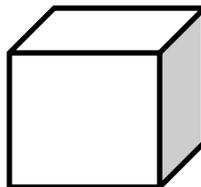
1. Find the area of the square at the right. (The answer will be in square inches.)



12 Inches

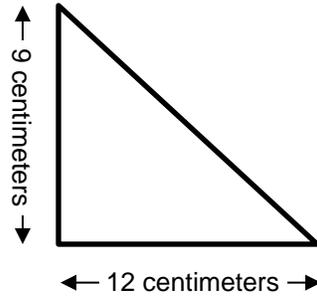
2. Find the perimeter of the square above. (The answer will be in inches.)

3. Find the volume of the cube at the right. (The answer will be in cubic inches.)

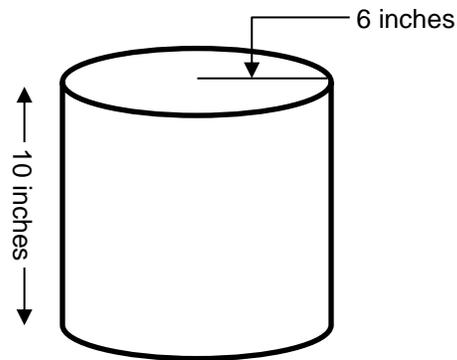


5 inches

4. Find the distance (d) traveled if you are going 38 miles per hour (r) for 2 hours (t). (The answer will be in miles.)
5. Find the area of a triangle where the base (b) is 12 centimeters and the height (h) is 9 centimeters. (The answer will be in square centimeters.)



6. Find the total cost of three children's jackets that cost \$49.95 each.
7. Find the volume of a cylinder if $\pi = 3.14$, the radius (r) is 6 inches, and the height (h) is 10 inches. (The units in the answer will be in cubic inches.)



Calculation & Formula Sheet

FORMULAS

AREA of a:

square	Area = side ²
rectangle	Area = length x width
parallelogram	Area = base x height
triangle	Area = ½ x base x height
trapezoid	Area = ½ x (base ₁ + base ₂) x height
circle	Area = π x radius ² ; π is approximately equal to 3.14.

PERIMETER of a:

square	Perimeter = 4 x side
rectangle	Perimeter = 2 x length + 2 x width
triangle	Perimeter = side ₁ + side ₂ + side ₃

CIRCUMFERENCE of a circle Circumference = π x diameter; π is approximately equal to 3.14.

VOLUME of a:

cube	Volume = edge ³
rectangular solid	Volume = length x width x height
square pyramid	Volume = ⅓ x (base edge) ² x height
cylinder	Volume = π x radius ² x height; π is approximately equal to 3.14.
cone	Volume = ⅓ x π x radius ² x height; π is approximately equal to 3.14.

COORDINATE GEOMETRY

distance between points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; (x_1, y_1) and (x_2, y_2) are two points in a plane.

Slope of a line = $\frac{y_2 - y_1}{x_2 - x_1}$; (x_1, y_1) and (x_2, y_2) are two points in a line.

PYTHAGOREAN RELATIONSHIP

$a^2 + b^2 = c^2$; a and b are legs and c the hypotenuse of a right triangle.

MEASURES OF CENTRAL TENDENCY

mean = $\frac{x_1 + x_2 + \dots + x_n}{n}$, where the x s are the values for which a Mean is desired, and n is the total number of values for x .

median = the middle value of an odd number of ordered scores, and halfway between the two middle values of an even number of ordered scores.

SIMPLE INTEREST

interest = principal x rate x time

DISTANCE

distance = rate x time

TOTAL COST

total cost = (number of units) x (price per unit)